CHAPTER 6 : Beyond elaticity : plasticity, yielding and ductility

6.1 Introduction and synopsis

The verb ‘to yield’ has two seemingly contradictory meanings. To yield under force is to submit to it, to surrender. To yield a profit has a different, more comfortable connotation: to bear fruit, to be useful. The *yield strength*, when speaking of a material, is the stress beyond which it becomes plastic. The term is well chosen: yield and the plasticity that follows can be profitable e it allows metals to be shaped, and it allows structures to tolerate impact and absorb energy. But the unplanned yield of the span of a bridge, or of the wing spar of an aircraft, or of the forks of your bicycle, spells disaster.

This chapter is about yield and plasticity. For that reason, it is mainly (but not wholly) about metals: it is the plasticity of iron and steel that made them the structural materials on which the Industrial Revolution was built, enabling the engineering achievements of the likes of Telford[1] and Brunel[2]. The dominance of metals in engineering, even today, derives from their ability to be rolled, forged, drawn and stamped.

6.2 Strength, ductility, plastic work and hardness: definition and measurement

Yield properties and ductility are measured using the standard tensile tests introduced in Chapter 4, with the materials taken to failure. Figures 6.1e6.3 show the types of stresseStrain behaviour observed in different material classes. The *yield stress* or *yield strength* s_y (or *elastic limit* s_{el}) e units: MPa or MN/m^2 e requires careful definition. For metals, the onset of plasticity is sometimes marked by a distinct kink in the stressestrain curve; if not, we identify s_y with the *0.2% proof stress* e that is, the stress at which the stressestrain curve for axial loading deviates by a permanent strain of 0.2% (this is the case shown in Figure 6.1). The elastic limit or yield strength is the same in tension and compression. When strained beyond the yield point, most metals *work harden*, causing the rising part of the curve, In tension, a maximum stress e the *tensile strength*, s_{ts} e is reached, followed by localised deformation (*necking*) and fracture.

---

[1] Thomas Telford (1757e1834), Scottish engineer, brilliant proponent of the suspension bridge at a time when its safety was a matter of debate. Telford may himself have had doubts e he was given to lengthy prayer on the days that the suspension chains were scheduled to take the weight of the bridge. Most of his bridges, however, still stand.

[2] Isambard Kingdom Brunel (1806e1859), perhaps the greatest engineer of the Industrial Revolution (c. 1760e1860) in terms of design ability, personality, power of execution and sheer willingness to take risks e the *Great Eastern*, for example, was five times larger than any previous ship ever built. He took the view that ‘great things are not done by those who simply count the cost’. Brunel was a short man and self-conscious about his height; he favoured tall top hats to make himself look taller.

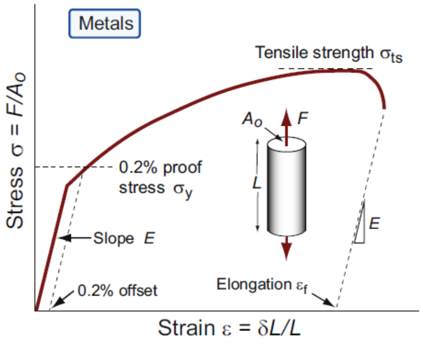


Figure 6.1 Stress–strain curve for a metal.

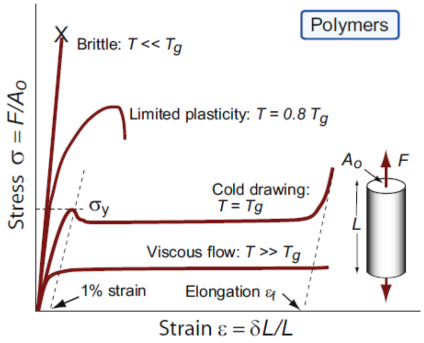


Figure 6.2 Stress–strain curve for a polymer.

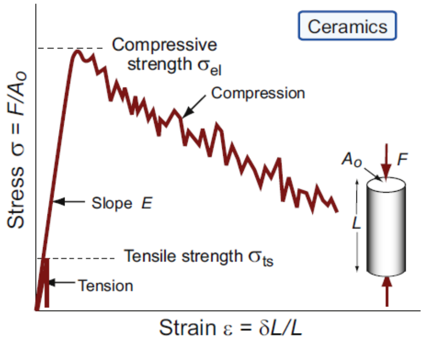


Figure 6.3 Stress–strain curve for a ceramic.

For polymers, σ_y is identified as the stress at which the stress–strain curve becomes markedly non-linear: typically, a strain of 1% (Figure 6.2). The behaviour beyond yield depends on the temperature relative to the glass temperature T_g. Well below T_g, most polymers are brittle. As T_g is approached, plasticity becomes possible until, at about T_g, thermoplastics exhibit *cold drawing*: large plastic extension at almost constant stress during which the molecules are pulled into alignment with the direction of straining, followed by hardening and fracture when alignment is complete. At still higher temperatures, thermoplastics become viscous and can be moulded; thermosets become rubbery and finally decompose. The yield strength σ_y of a polymer–matrix composite is also best defined by a set deviation from linear elastic behaviour, typically 0.5%. Composites that contain aligned fibres (and this includes natural composites like wood) are a little weaker (up to 30%) in compression than tension because the fibres buckle on a small scale.

Ceramics and glasses are brittle at room temperature (Figure 6.3). They do have yield strengths, but these are so enormously high that, in tension, they are never reached: the materials fracture first. Even in compression, ceramics and glasses crush before they yield. To measure their yield strengths, special tests that suppress fracture are needed. We will return to the fracture-controlled strength of ceramics in Chapter 8. For now, it is useful to have a practical measure of the strength of ceramics to allow their comparison with other materials. The measure used here is the *compressive crushing strength*, and since it is not a yield point but is still the end of the elastic part of the stress–strain curve, we call it the *elastic limit* and give it the symbol σ_{el}.

Returning then to ductile materials, the *plastic strain*, ε_{pl}, is the permanent strain resulting from plasticity; thus, it is the total strain ε_{tot} minus the recoverable elastic part:

$$\varepsilon_{pl} = \varepsilon_{\text{tot}} - \frac{\sigma}{E} \tag{6.1}$$

The *ductility* is a measure of how much plastic strain a material can tolerate. It is measured in standard tensile tests by the *elongation* ε_f (the plastic tensile strain at fracture) expressed as a percentage (Figures 6.1 and 6.2). Strictly speaking, ε_f is not a material property because it depends on the sample dimensions – the values that are listed in handbooks and in the Cambridge Engineering Selector software are for a standard test geometry – but it remains useful as an indicator of the ability of a material to be deformed.

In Chapter 4, the area under the elastic part of the stress–strain curve was identified as the elastic energy stored per unit volume ($\sigma_y^2/2E$). Beyond the elastic limit, *plastic work* is done in deforming a material permanently by yield or crushing. The increment of plastic work done for a small permanent extension or compression dL under a force F, per unit volume $V = A_o L_o$, is

$$dW_{pl} = \frac{FdL}{V} = \frac{F}{A_o}\frac{dL}{L_o} = \sigma d\varepsilon_{pl}$$

Thus, the plastic work per unit volume at fracture is

$$W_{pl} = \int_0^{\varepsilon_f} \sigma d\varepsilon_{pl} \tag{6.2}$$

which is just the area under the whole stress–strain curve (minus the elastic unloading at the end). This is important in energy-absorbing applications, such as the crash boxes in the front of cars or roadside crash barriers.

Tensile and compression tests are not always convenient: the component or sample is destroyed. The hardness test (Figure 6.4) is a *non-destructive test* that avoids this problem. A pyramidal diamond or a hardened steel ball is pressed into the surface of the material. This leaves a tiny permanent indent, the width of which is measured with a microscope and can be converted to a projected area A perpendicular to the load. The indent means that plasticity has occurred, and the hardness H measures the resistance to plastic indentation. By the simplest definition, hardness is defined as:

$$H = \frac{F}{A} \tag{6.3}$$

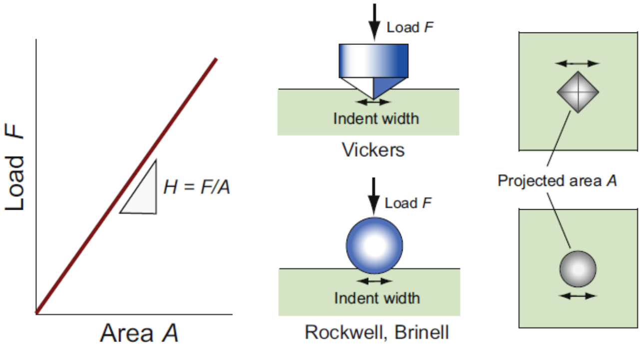


Figure 6.4 The hardness test. The Vickers test uses a diamond pyramid; the Rockwell and Brinell tests use a steel sphere, producing the indent shapes shown on the right.

In practice, corrections may also be made to convert the projected area A to the actual area of contact between indenter and material – this is a small correction, as the indentations are shallow compared to their width. The plastic region under the indenter is surrounded by material that has not deformed, and this pushes inwards and constrains the plastic deformation so that H is larger than the (uniaxial) yield strength σ_y – in practice, it is about $3\sigma_y$. Strength, as we have seen, is measured in units of MPa, and since H is also a force per unit area, it would be logical to measure it in MPa too, but convention dictates otherwise in this case. A commonly used scale, that of *Vickers hardness* (symbol H_v), has units of kg/mm^2, which is equivalent to H in MPa divided by g (=9.81 m/s^2, the acceleration due to gravity), with the result that

$$H_v \approx \frac{3\,\sigma_y}{g} \approx \frac{\sigma_y}{3} \tag{6.4}$$

The conversion between hardness scales and the corresponding approximate yield strength are shown in Figure 6.5. The hardness test has the advantage of being non-destructive, so strength can be measured without destroying the component, and it requires only a tiny volume of material. But the information it provides is less accurate and less complete than the tensile test, so it is not used to provide critical design data.

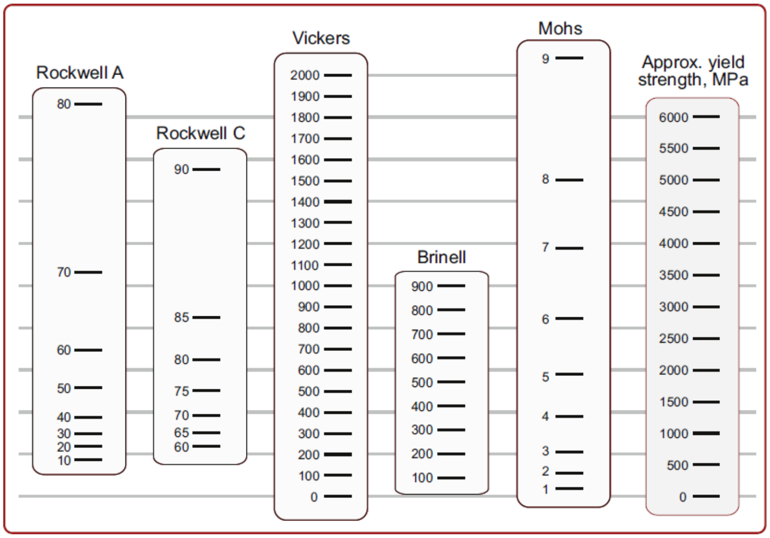


Figure 6.5 Common hardness scales compared with the yield strength.

True stress and true strain The graphs of Figures 6.1–6.3 strictly show the *nominal stress–nominal strain* responses; that is, the area and length used are the original values at the start of the test. But once materials yield, they can change dimensions significantly. The *true stress*, σ_t, takes account of the current dimensions, so in tension, for example (equation (4.1))

$$\sigma_t = \frac{F}{A} \tag{6.5}$$

To relate this to the nominal stress, which we will refer to as σ_n, we note that in plastic deformation, the volume of the sample is conserved, for reasons discussed later in this chapter; that is

$$\text{Volume} = A_o L_o = AL \tag{6.6}$$

where A and L are the current area and length of the test sample, and were initially A_o and L_o. Hence, the nominal stress is

$$\sigma_n = \frac{F}{A_o} = \frac{F}{A}\left(\frac{L_o}{L}\right) \tag{6.7}$$

Returning to the definition of nominal strain, ε_n (equation (4.3))

$$\varepsilon_n = \frac{\delta L}{L_o} = \left(\frac{L - L_o}{L_o}\right) = \left(\frac{L}{L_o}\right) - 1 \tag{6.8}$$

Combining with equations (6.5) and (6.7) gives the true stress as

$$\sigma_t = \sigma_n(1 + \varepsilon_n) \tag{6.9}$$

True strain, ε_t, is a bit more involved. An incremental change in length, dL, gives an incremental strain relative to the current length L. To find the cumulative true strain, we therefore integrate these increments over the full extension from L_o to L, giving

$$\varepsilon_t = \int_{L_o}^{L} \frac{dL}{L} = \ln\left(\frac{L}{L_o}\right) = \ln(1 + \varepsilon_n) \tag{6.10}$$

To see how different the true and nominal quantities are in practice, let's do an example.

The example shows that for the elastic regime and small plastic strains, the difference between nominal and true stresses and strains is negligible. This explains the effectiveness of using nominal values to define key properties such as the yield stress. By the end of the tensile test in a metal, the discrepancy between the two is typically 20%.

What about compressive true stress and strain? Now the cross-sectional area increases with plastic deformation (to conserve volume). Equations (6.5) to (6.10) all hold, so long as care is taken with the sign of the applied force F and the strain increments dL, which are negative. Figure 6.6 shows nominal and true stress–strain curves superimposed, for both compression testing and for the stable part of a tensile test up to the tensile strength. As observed in Example 6.2, in tension, the true stress is higher than nominal, whereas for strain, the reverse is true. In compression, nominal stress exceeds true, and again the reverse for strain. Figure 6.6 shows one benefit of true stress–strain: the curves are identical in shape for tension and compression. The compressive side also extends to larger values of stress and strain, as the instability of necking in tension is avoided. The true stress–strain behaviour is most useful for *metal forming*, where plastic strains of the order of 50% or more are common. And some forming processes use multiple passes, with several plastic deformations one after another. In the Exercises at the end of the chapter, you can show that the total strain in this case can simply be found by adding up the true strains, but this is not the case for nominal strains.

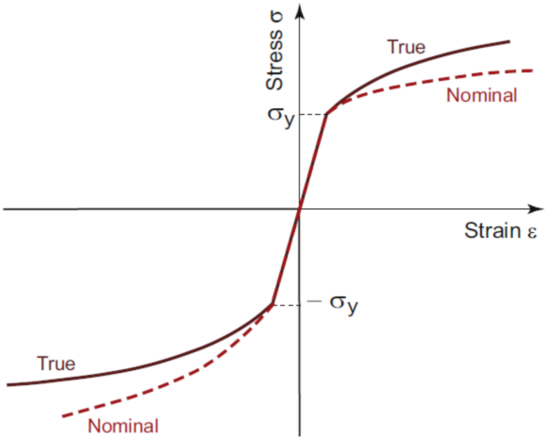


Figure 6.6 Nominal and true stress–strain curves for a ductile metal, in tension and compression.

6.3 The big picture: charts for yield strength

Strength can be displayed on material property charts. Two are particularly useful: the strength–density chart and the modulus–strength chart.

The strength–density chart Figure 6.7 shows the yield strength σ_y or elastic limit σ_{el} plotted against density ρ. The range of strength for engineering materials, like that of the modulus, spans about 6 decades: from less than 0.01 MPa for foams, used in packaging and energy-absorbing systems, to 10^4 MPa for diamond, exploited in diamond tooling for machining and as the indenter of the Vickers hardness test. Members of each family again cluster together and can be enclosed in envelopes, each of which occupies a characteristic part of the chart.

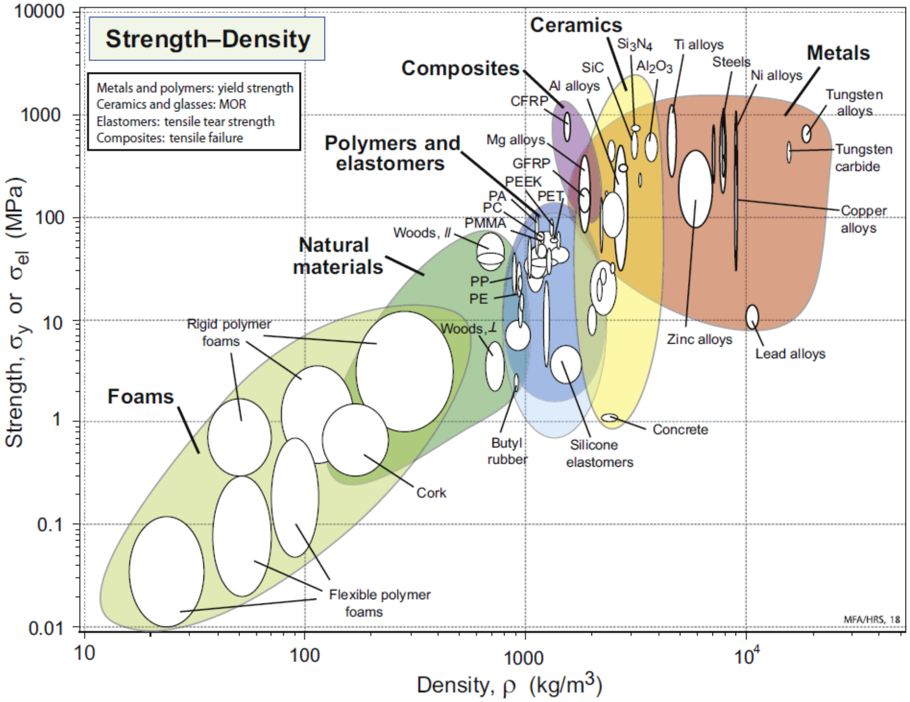


Figure 6.7 The strength–density property chart.

Comparison with the modulus–density chart (Figure 4.7) reveals some marked differences. The modulus of a solid is a well-defined quantity with a narrow range of values. The strength is not. The strength range for a given class of metals, such as stainless steels, can span a factor of 10 or more, while the spread in stiffness is at most 10%. Since density varies very little (Chapter 4), the strength bubbles for metals are long and thin. The wide ranges for metals reflect the underlying physics of yielding, and present designers with an opportunity for manipulation of the strength by varying composition and process history. Both are discussed later in this chapter.

Polymers cluster together with strengths between 10 and 100 MPa, and the modulus and strength vary by a similar amount for a given polymer. The composites carbon fibre-reinforced polymer (CFRP) and glass fibre-reinforced polymer (GFRP) have strengths that lie between those of polymers and ceramics, as we might expect since they are mixtures of the two. The analysis of the strength of composites is not as straightforward as for modulus in Chapter 4, though the same bounds (with strength replacing modulus) generally give realistic estimates.

The modulus–strength chart Figure 6.8 shows Young's modulus, E, plotted against yield strength, σ_y or elastic limit, σ_{el}. This chart allows us to examine a useful material characteristic, the yield strain, σ_y/E, meaning the strain at which the material ceases to be linearly elastic. On log axes, contours of constant yield strain appear as a family of straight parallel lines, as

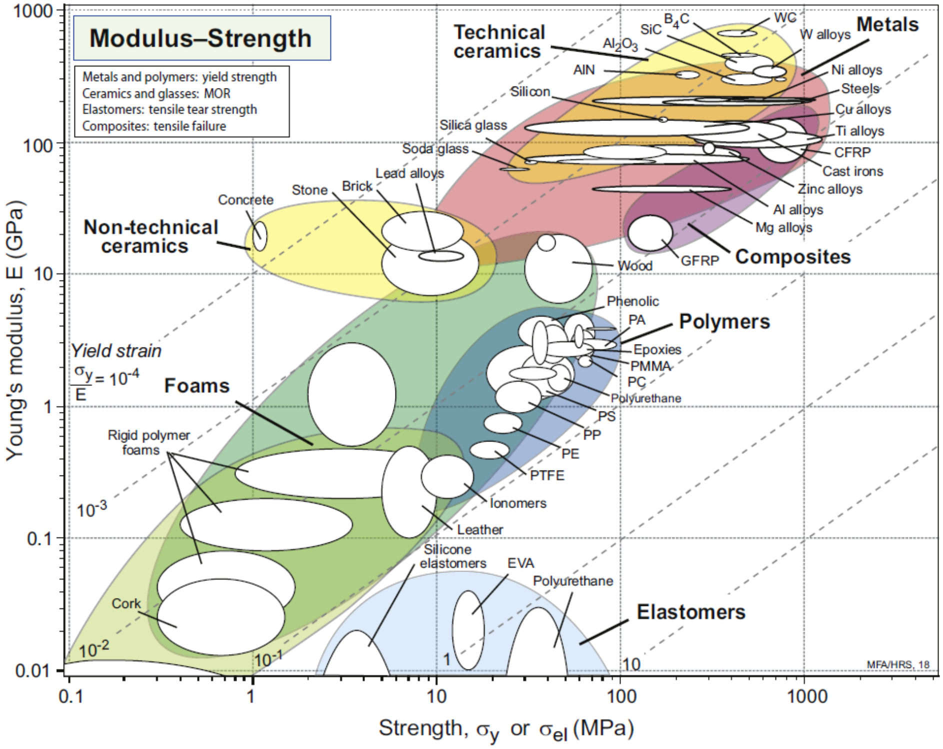


Figure 6.8 The Young's modulus–strength chart. The contours show the strain at the elastic limit, σ_y/E.

shown in Figure 6.8. Engineering polymers have large yield strains, between 0.01 and 0.1; the values for metals are at least a factor of 10 smaller. Composites and woods lie on the 0.01 contour, as good as the best metals. Elastomers, because of their exceptionally low moduli, have values of σ_y/E in the range 1–10, much larger than any other class of material.

This chart has many other applications, notably, in selecting materials for springs, elastic diaphragms, flexible couplings and snap-fit components. We explore these in Chapter 7.

6.4 Drilling down: the origins of strength and ductility

Perfection: the ideal strength The bonds between atoms, like any other spring, have a breaking point. Figure 6.9 shows a stress–strain curve for a single bond. Here an atom is assumed to occupy a cube of side length a_o (as in Chapter 4) so that a force F corresponds to a stress F/a_o^2. The force stretches the bond from its initial length a_o to a new length a, giving a strain $(a - a_o)/a_o$. When discussing the modulus in Chapter 4, we focused on the initial, linear part of this curve, with a slope equal to the modulus, E. Stretched farther, the curve passes through a maximum and falls to zero as the atoms lose their interaction altogether. The peak is the bond strength – if you pull harder than this, it will break. The same is true if you shear it rather than pull it.

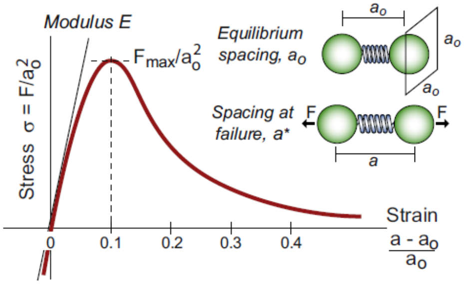


Figure 6.9 The stress–strain curve for a single atomic bond (it is assumed that each atom occupies a cube of side a_o).

The distance over which inter-atomic forces act is small – a bond is broken if it is stretched to more than about 10% of its original length. So the force needed to break a bond is roughly:

$$F = \frac{S a_o}{10} \tag{6.11}$$

where S, as before, is the bond stiffness. On this basis, the *ideal strength* of a solid should therefore be roughly

$$\sigma_{ideal} = \frac{F_{max}}{a_o^2} = \frac{S}{10 a_o} = \frac{E}{10}$$

remembering that $E = S/a_o$, equation (4.26). Hence:

$$\frac{\sigma_{\text{ideal}}}{E} = \frac{1}{10} \tag{6.12}$$

This doesn't allow for the curvature of the force–distance curve; more refined calculations give a ratio of 1/15.

Figure 6.10 shows σ_y/E for metals, polymers and ceramics. None achieves the ideal value of 1/15; most don't even come close. To understand why, we need to look at the microstructure and imperfections in each material class.

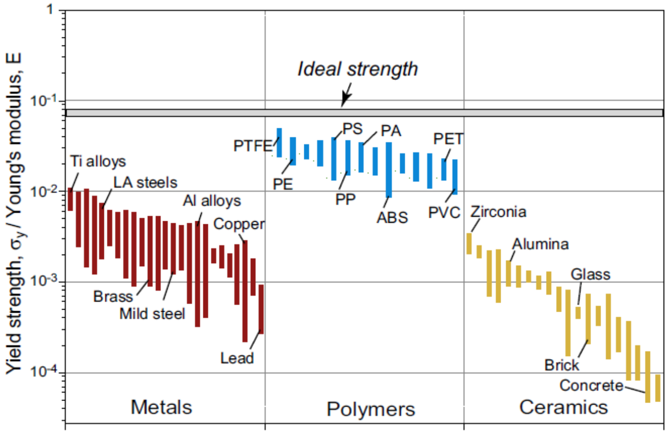


Figure 6.10 The ideal strength is predicted to be about $E/15$, where E is Young's modulus. The figure shows σ_y/E with a shaded band at the ideal strength.

Strength of polymers Figure 6.2 showed a range of stress–strain responses in polymers. Recall first that in Chapter 4, the glass transition temperature T_g was defined, being the temperature at which the inter-molecular hydrogen bonds melt. This is usually associated with a marked drop in the material stiffness, depending on whether the polymer is amorphous, semi-crystalline or cross-linked. Not surprisingly, the strength of a polymer also depends strongly on whether the material is above or below the glass transition, and a number of different failure mechanisms can occur.

At low temperatures, meaning below about 0.75 T_g, polymers are elastic–brittle, like ceramics, and fail by propagation of a dominant flaw (see Chapter 8). Above this temperature, they become plastic. When pulled in tension, the chains slide over each other, unravelling, so that they become aligned with the direction of stretch, as in Figure 6.11(a), a process called *drawing*. It is harder to start drawing than to keep it going, so the zone where it starts draws down completely before propagating farther along the sample, leading to profiles like that shown in the figure. The drawn material is stronger and stiffer than before, by a factor of about 8, giving drawn polymers exceptional properties, but because you can only draw fibres, or sheet (by pulling in two directions at once), the geometries are limited.

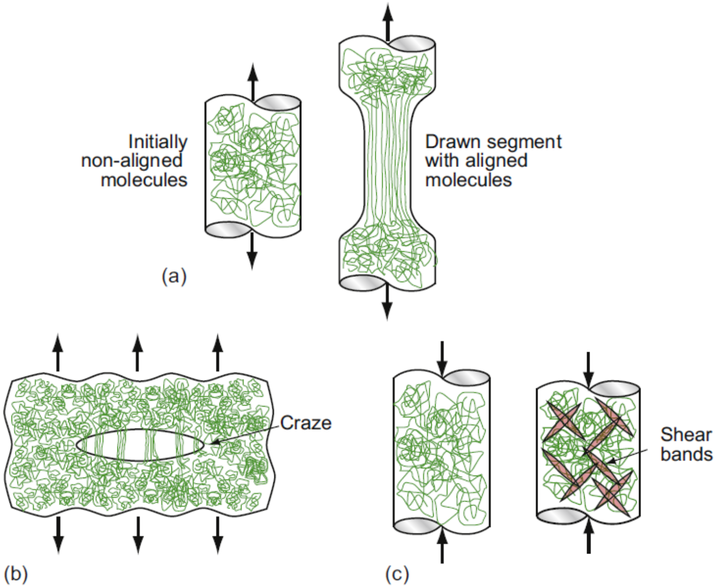


Figure 6.11 (a) Cold drawing — one of the mechanisms of deformation of thermoplastics. (b) Crazing — local drawing across a crack. (c) Shear banding.

Many polymers, among them PE, PP and nylon, draw at room temperature. Others with higher glass temperatures, such as PMMA, do not, although they draw well at higher temperatures. At room temperature, they *craze*. Small crack-shaped regions within the polymer draw down. Because the crack has a larger volume than the polymer that was there to start with, the drawn material ends up as ligaments of aligned segments of molecules that link the craze surfaces, as in Figure 6.11(b). Crazes scatter light, so their presence causes whitening, easily visible when cheap plastic articles are bent. If stretching is continued, one or more crazes develop into proper cracks, and the sample fractures.

While crazing limits ductility in tension, large plastic strains may still be possible in compression by *shear banding* (Figure 6.11(c)). These bands allow blocks of material to either side of the band to shear with respect to one another, without the material coming apart. Continued compression causes the number of shear bands to increase, giving increased overall strain.

The upshot of all this is that polymers do have a strength that is not very far below their ideal value (Figure 6.10). But we shouldn't get too excited — their moduli are much lower than metals to begin with, but it does mean that polymers are not so far below metals when compared on strength (an observation you can make for yourself on the modulus–strength property chart of Figure 6.8).

Crystalline imperfection: defects in metals and ceramics Crystals contain imperfections of several kinds — Figure 6.12. All crystals contain point defects called *vacancies*, shown in (a): sites at which an atom is missing. They play a key role in diffusion, creep and sintering (Chapter 13), but we don't need them for the rest of this chapter because they do not influence strength.

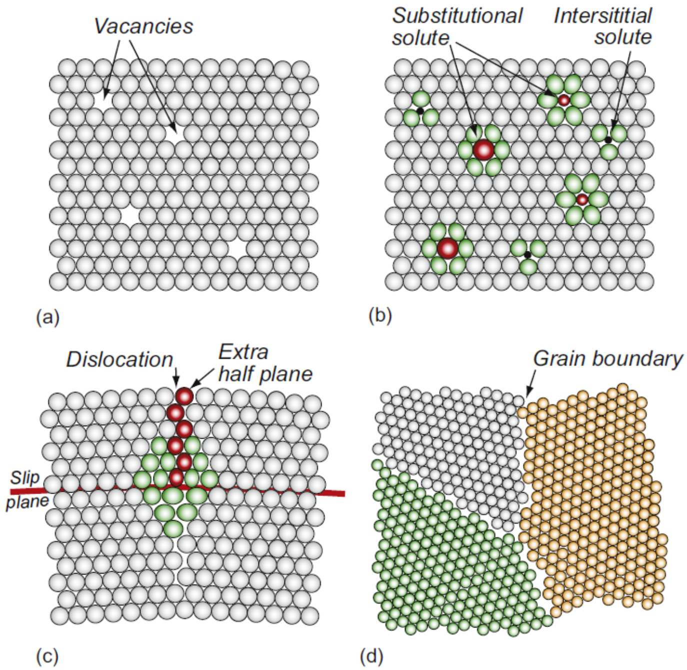


Figure 6.12 Defects in crystals. (a) Vacancies — missing atoms. (b) Foreign (solute) atoms in interstitial and substitutional sites. (c) A dislocation — an extra half-plane of atoms. (d) Grain boundaries.

No crystal is made up of a single pure element. Other elements are always present at some concentration — either as impurities inherited from the process by which the material was made, or they are deliberately added, creating *alloys*: a material in which two or more elements are mixed. If the foreign elements 'dissolve' and are spread out atomically in the crystal — like salt in water — then we have a *solid solution*. Figure 6.12(b) shows both a *substitutional solid solution* (the dissolved atoms replace those of the host) and an *interstitial solid solution* (the dissolved atoms squeeze into the spaces or 'interstices' between the host atoms). The dissolved atoms or solute rarely have the same size as those of the host material, so they distort the surrounding lattice. The red atoms here are substitutional solute, some bigger and some smaller than those of the host; the cages of host atoms immediately surrounding them, shown green, are distorted. Only the smallest atoms dissolve *interstitially*, like the black atoms in the figure, but these too distort the surrounding lattice. So solute causes local distortion; this distortion is one of the reasons that alloys are stronger than pure materials, as we shall see in a moment.

Now to the key player, portrayed in Figure 6.12(c): the *dislocation*. We will see shortly how these are responsible for making metals soft and ductile, well below their ideal strength. 'Dislocated' means 'out of joint', and this is not a bad description of what is happening here.

The upper part of the crystal has one ‘half-plane’ of atoms than the lower part – the region around the bottom of this half-plane is a dislocation. They are actually *line defects*, extending perpendicular to the view shown in the figure. Dislocations distort the lattice – here the green atoms are the most distorted – and because of this, they have elastic energy associated with them. If they cost energy, why are they there? To grow a perfect crystal just one cubic centimetre in volume from a liquid or vapour, about 10^{23} atoms have to find their proper sites on the perfect lattice, and the chance of this happening is just too small. Even with the greatest care in assembling them, all crystals contain vacancies and dislocations.

Most crystalline solids are polycrystalline, made up of very many tiny crystals, or *grains*. Figure 6.12(d) shows three perfect, but differently oriented, grains meeting, which leads to interfaces between the grains where the packing is irregular. These planar defects are called *grain boundaries*. In this sketch, the atoms of the three crystals have been given different colours to distinguish their packing orientations, but they are all the same atoms. In reality, grain boundaries form in pure materials (when all the atoms are the same) and in alloys (when the mixture of atoms in one grain may differ in chemical composition from those of the next).

So the outward perfection of the steel of a precision machine tool, or of the polished case of a gold watch, is an illusion: both metals are full of defects of the types shown in Figure 6.12. Between them, they explain diffusion of atoms in solids, and the *microstructure-sensitive properties*: strength, ductility, electrical resistance, thermal conductivity and much more. At the heart of metal alloying and processing is the direct manipulation of these defects to control properties – in this chapter, we learn how strength depends on microstructure; in Chapter 19 and *Guided Learning Unit* 2, we add the context of processing.

So defects in crystals are influential. For the rest of this section, we focus on dislocations, as these control the plastic behaviour of metals. First we need to understand them to explain why the strength of metals is below ideal (Figure 6.10), and then in Section 6.5, we address how strength is increased by forcing dislocations to interact with the other defects.

Ceramics can also deform plastically – but only if they are very hot (and generally they also need to be in compression). But their tensile strength shown in Figure 6.10 is also well below yield. To explain this, we need *fracture mechanics*, and a different type of defect: *cracks*. Chapter 8 is where we will tackle fracture.

Dislocations and plastic flow Recall that the strength of a perfect crystal computed from inter-atomic forces gives an ‘ideal strength’ around $E/15$ (where E is the modulus). In reality, the strengths of engineering alloys are nothing like this big; often they are barely 1% of it. This was a mystery until the 1930s when an Englishman, G. I. Taylor[3], and a Hungarian, Egon Orowan[4], realized that a ‘dislocated’ crystal could deform at stresses far below the ideal. So what is a dislocation, and how does it enable deformation?

---

[3] Geoffrey (G. I.) Taylor (1886–1975), known for his many fundamental contributions to aerodynamics, hydrodynamics and to the structure and plasticity of metals – it was he, with Egon Orowan, who realised that the ductility of metals implied the presence of dislocations. One of the greatest of contributors to theoretical mechanics and hydrodynamics of the 20th century, he was also a supremely practical man – a sailor himself, he invented (among other things) the anchor used by the Royal Navy.

[4] Egon Orowan (1901–1989), Hungarian/US physicist and metallurgist, who, with G. I. Taylor, realised that the plasticity of crystals could be understood as the motion of dislocations. In his later years, he sought to apply these ideas to the movement of fault lines during earthquakes.

Figure 6.13(a) shows how to make a dislocation. Imagine that the crystal is cut along a horizontal atomic plane up to the line shown as ⊥ − ⊥, and the top part above the cut is then slipped across the bottom part by one full atom spacing. Then most of the atoms either side of the cut can be reattached across the slip plane, except at the very end of the cut, where there is a misfit in the atom configuration, as shown in Figure 6.13(b) for a simple cubic lattice packing. The problem is that there is now effectively an extra half-plane of atoms in the top part of the crystal, with its lower edge along the ⊥ − ⊥ line. This is the *dislocation line* – the line separating the part of the plane that has slipped from the part that has not. This particular configuration is called an *edge dislocation* because it is formed by the edge of the extra half-plane, represented by the symbol ⊥.

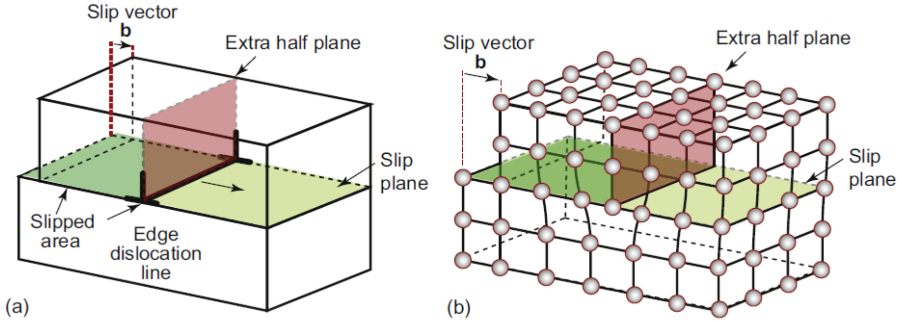


Figure 6.13 (a) Making a dislocation by cutting, slipping and rejoining bonds across a slip plane. The slip vector **b** is perpendicular to the edge dislocation line ⊥−⊥. (b) The atom configuration round an edge dislocation in a simple cubic crystal.

Dislocations control plastic strain, since if a dislocation moves across its slip plane, then it makes the material above the plane slide one atomic spacing relative to that below. Figure 6.14 shows how this happens. At the top is a perfect crystal. In the central row, a shear stress τ is acting parallel to the slip planes, and this drives an edge dislocation in from the left; it sweeps through the crystal and exits on the right. Note that no individual atom moves very far as the dislocation moves – the bonds round the dislocation are stretched to and fro to allow the pattern of misfitting atoms to move along the slip plane. By the end of the process, the upper part has slipped relative to the lower part by a slip step **b**, known as the Burger's vector. The result is the shear strain γ shown at the bottom. Note that in this configuration, the shear stress is perpendicular to the edge dislocation, which moves in the direction of the stress, and the resulting slip step **b** is in the direction of the shear stress.

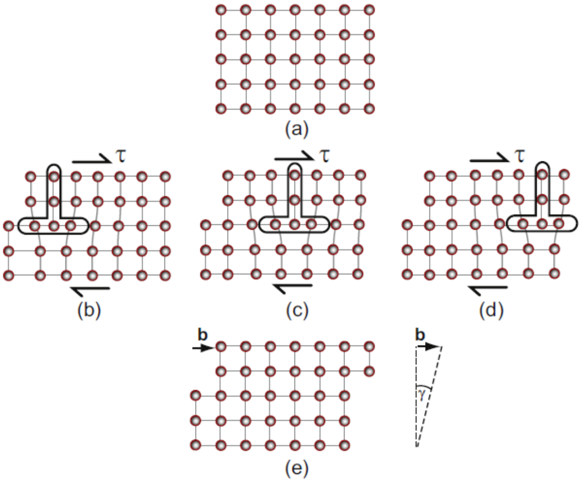


Figure 6.14 An initially perfect crystal is shown at (a). A shear stress τ drives the passage of a dislocation across the slip plane, shown in the sequence (b), (c), and (d), and shears the upper part of the crystal over the lower part by the slip vector **b**, leading at (e) to a shear strain γ.

There is another way to make a dislocation in a crystal. After making the cut in Figure 6.13(a), the upper part of the crystal can be displaced *parallel* to the edge of the cut rather than *normal* to it, as in Figure 6.15(a). That too creates a dislocation, but one with a different configuration of misfitting atoms along its line – more like a corkscrew – and for this reason, it is called a *screw dislocation*. We don't need to try and visualise the details of its structure; it is enough to know that its properties are similar to those of an edge dislocation, with the following distinction: the driving shear stress is now *parallel* to the screw dislocation, which sweeps sideways through the crystal, again moving normal to itself, but the slip step **b** remains in the direction of applied shear stress. In fact, dislocations also form curves as they move over a slip plane, in which case they are called *mixed dislocations*, meaning that they are made up of little steps of edge and screw. The 'pure' edge and screw configurations are then found at two positions on the dislocation at 90° to one another – as shown in Figure 6.15(b). Once we've seen all this, it is easiest to forget about where the atoms are and just to think of dislocations as flexible line defects gliding over the slip planes under the action of an applied shear stress. It doesn't matter whether the dislocation is edge, screw or mixed: it always moves perpendicular to itself, and every part of it produces the same slip vector **b** in the direction of the shear stress – the dislocation line is just the boundary on the plane up to which point which a fixed displacement **b** has occurred.

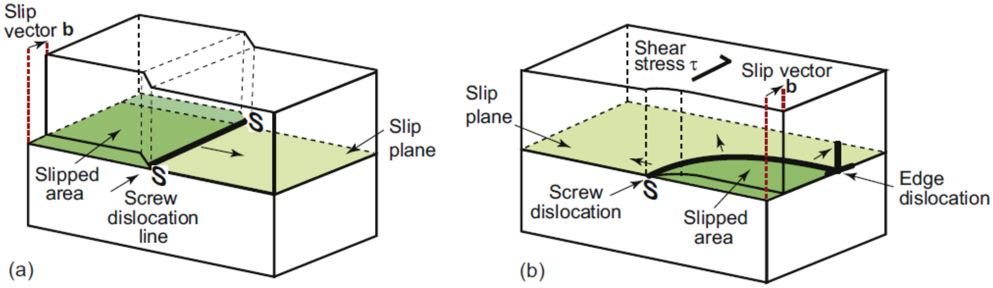


Figure 6.15 (a) A screw dislocation, with the slip vector **b** parallel to the dislocation line S–S; (b) a curved, mixed dislocation, connecting a pure edge to a pure screw dislocation.

The dislocation is key to understanding why metals have a strength that is so far below their ideal strength: it is far easier to move a dislocation through a crystal, stretching and remaking bonds only locally along its line as it moves, than it is to simultaneously break *all* the bonds in the plane before remaking them. It is like moving a heavy carpet by pushing a fold across it rather than sliding the whole thing at one go. In real crystals, it is easier to make and move dislocations on some planes than on others. The preferred planes are called *slip planes*, and the preferred directions of slip in these planes are called *slip directions*. We have seen these previously in the close-packed hexagonal, face-centred cubic (FCC) and body-centred cubic structure unit cells of Figure 4.30 – dislocations prefer planes and directions that are close-packed, or nearly so.

Single slip steps are tiny – one dislocation produces a displacement of about 10^{-10} m. But if large numbers of dislocations traverse a crystal, moving on many different planes, the shape of a material changes at the macroscopic length scale. Figure 6.16 shows just two dislocations traversing a sample loaded in tension, and then another two on a different pair of slip planes. The slip steps (here very exaggerated) cause the sample to get a bit thinner and longer. Repeating this millions of times on many planes gives the large plastic extensions observed in practice. Since none of this changes the average atomic spacing, the volume remains unchanged. So this is how materials deform plastically at stresses well below the ideal strength – but note the crucial advantages: the material remains fully intact without any loss of strength as its shape changes significantly. This ductility is essential for safe design – we can live with a bit of unintended strain, but not with everything being brittle – and it also enables us to shape components from lumps of metal in the solid state.

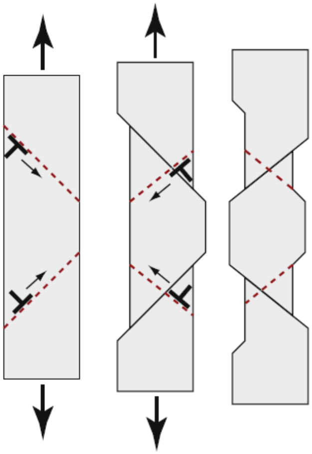


Figure 6.16 Dislocation motion leads to strain but conserves volume.

Why does a shear stress make a dislocation move? A shear stress is needed to move dislocations because crystals resist their motion with a friction-like resistance *f* per unit length – we will examine its origins in a moment. But for yielding to take place, the shear stress must overcome the resistance *f*.

Imagine that one dislocation moves right across a slip plane, traveling the distance L_2, as in Figure 6.17. In doing so, it shifts the upper half of the crystal by a distance b relative to the lower half. The shear stress τ acts on an area L_1L_2, giving a shear force $F_s = \tau L_1 L_2$ on the surface of the block. If the displacement parallel to the block is b, the force does work

$$W = \tau L_1 L_2 b \tag{6.13}$$

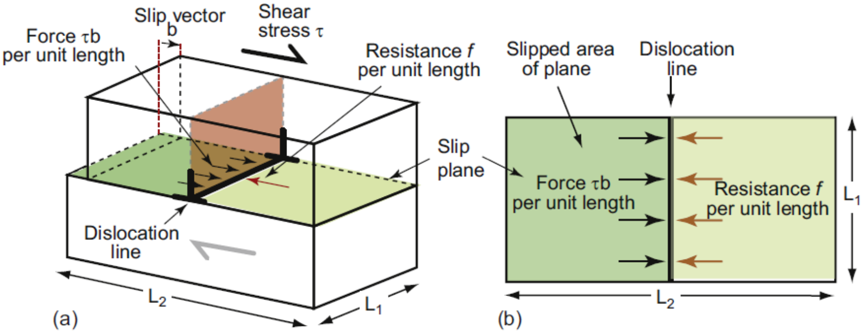


Figure 6.17 The force on a dislocation. (a) perspective view; (b) plan view of slip plane.

This work is done against the resistance force f per unit length, or $f\,L_1$ on the length L_1, and it does so over a displacement L_2 (because the dislocation line moves this far against f), giving a total work against f of fL_1L_2. Equating this to the work W done by the applied stress τ gives

$$\tau b = f \tag{6.14}$$

This result holds for any dislocation – edge, screw or mixed. So, provided the shear stress τ exceeds the value f/b, it will make dislocations move and cause the crystal to shear.

Line tension The atoms near the 'core' of a dislocation are displaced from their proper positions, as shown by green atoms back in Figure 6.12(c), and thus they have higher potential energy. To keep the potential energy of the crystal as low as possible, the dislocation tries to be as short as possible – it behaves as if it had a *line tension*, T, like an elastic band. The tension can be calculated but it needs advanced elasticity theory to do it (the books listed under 'Further Reading' give the analysis). We just need the answer. It is that the line tension T, an energy per unit length (just as a surface tension is an energy per unit area), is

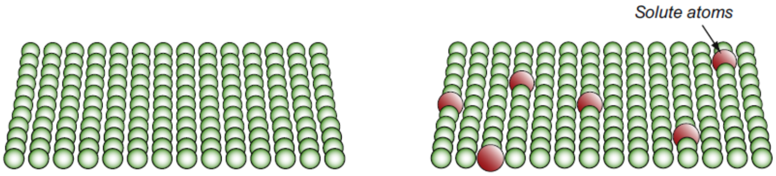
$$T = \frac{1}{2}Gb^2 \tag{6.15}$$

where G is the shear modulus (since dislocations are associated with local lattice straining in shear), and b is the magnitude of the Burger's vector. Recall that for metals, G scales with Young's modulus E, with $G \approx 3/8\,E$ (Chapter 4). The line tension has an important bearing on the way in which dislocations interact with obstacles, as we shall see in a moment. At first, it is perhaps surprising that we explain *plastic* deformation via dislocation motion, which turns out to depend on an *elastic* property – the shear modulus, G. But it derives from the fact that dislocations strain the lattice elastically by small (elastic) changes in the atomic spacing. This local elastic distortion enables the dislocations to move large distances, leading to large plastic strains.

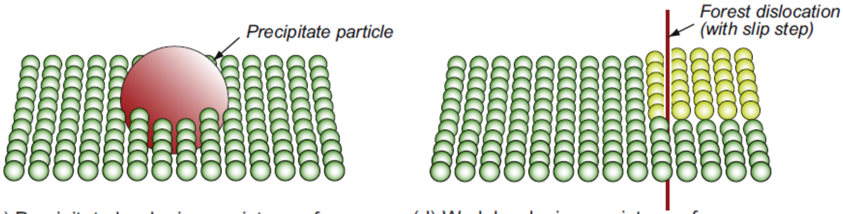
The lattice resistance Where does the resistance to slip, f, come from? There are several contributions. Consider first the *lattice resistance*, f_i: the intrinsic resistance of the crystal structure to plastic shear. Plastic shear, as we have seen, involves the motion of dislocations. Pure metals are soft because the non-localised metallic bond does little to obstruct dislocation motion, whereas ceramics are hard because their more localised covalent and ionic bonds (which must be stretched and reformed when the lattice is sheared) lock the dislocations in place. When the lattice resistance is high, as in ceramics, further hardening is superfluous – the problem becomes that of suppressing fracture. On the other hand, when the lattice resistance f_i is low, as in metals, the material needs to be strengthened by introducing obstacles to slip. This is done by adding alloying elements to give *solid solution hardening* (f_{ss}), and using precipitates or dispersed particles to give *precipitation hardening* (f_{ppt}), while other dislocations give what is called *work hardening* (f_{wh}), and polycrystals also have *grain boundary hardening* (f_{gb}). These techniques for manipulating strength are central to alloy design. We look at them more closely in the next section.

6.5 Manipulating strength

Strengthening metals: dislocation pinning The way to make crystalline materials stronger is to make it harder for dislocations to move. As we have seen, dislocations move in a pure crystal when the force τb per unit length exceeds the lattice resistance f_i. There is little we can do to change this – it is an intrinsic property like the modulus E. Other strengthening mechanisms add to it, and here there is scope for manipulation. Figure 6.18 shows views of a slip plane containing obstacles from the perspective of an advancing dislocation; each strengthening mechanism presents a different obstacle course. In the perfect lattice shown in (a), the only resistance is the intrinsic strength of the crystal; solution hardening, shown in (b), introduces atom-size obstacles to motion; precipitation hardening, shown in (c), presents larger obstacles; and in work hardening, shown in (d), the slip plane becomes stepped and threaded with 'forest' dislocations.



(a) Perfect lattice, resistance f_i (b) Solution hardening, resistance f_{ss}



(c) Precipitate hardening, resistance f_{ppt} (d) Work hardening, resistance f_{wh}

Figure 6.18 A 'dislocation-eye' view of the slip plane across which it must move.

Figure 6.19(a) shows what happens when dislocations encounter obstacles, which act as *pinning points*. From equation (6.14), the force on the dislocation between two pinning points of spacing L is $F = \tau bL$. From the figure, there is one obstacle for each length L of dislocation, so the force on each obstacle is also $F = \tau bL$. To keep the dislocation moving, an increase in the shear stress is needed to bow out the dislocation through the gap. The *bowing angle* θ and applied shear stress τ increase until the force F can overcome the *pinning force* p that the obstacle can apply on the dislocation. Figsure 6.19(b) and (c) show two ways in which dislocations escape from pinning points: 'weak' obstacles are passed when the bowing angle θ lies between 0° and 90°; for 'strong' obstacles, the bowing angle reaches the limiting value

$\theta = 90°$, and the dislocation escapes by other means – the arms of the dislocations link up and the dislocation advances, but leaving a dislocation loop around the obstacle (Figure 6.19(c)). A helpful way to think about dislocation pinning is to recall that dislocations have a 'line tension' trying to make the dislocation as short as possible. As the dislocation bows out, the line tension 'pulls' against the obstacle, with the maximum pull being exerted when $\theta = 90°$.

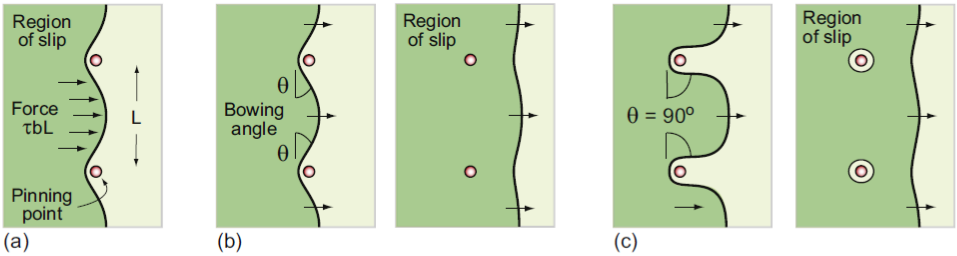


Figure 6.19 Dislocation pinning: (a) force on dislocation between pinning points; (b, c) dislocation bowing and escape from weak and strong obstacles, respectively.

In both weak and strong cases in Figure 6.19, the dislocation escapes when $F = p$. Hence, the additional contribution to the shear stress τ needed to overcome the pinning points is

$$\Delta\tau = \frac{p}{bL} \tag{6.16}$$

Dislocation pinning is an elastic effect, locally distorting the atoms around the dislocation. As a result, the pinning force p, for each type of obstacle in a given material, scales with the energy per unit length (or line tension), of the dislocation, $Gb^2/2$ (equation (6.15)). Combining this with equation (6.16), the shear stress τ needed to force the dislocation through a field of obstacles therefore has the form

$$\tau = \alpha\frac{Gb}{L} \tag{6.17}$$

where α is a dimensionless constant characterizing the obstacle strength. Take a close look at this equation: G and b are material constants, so in order to manipulate the strength, the parameters that we can control are: α (the obstacle strength) and L (the obstacle spacing). Armed with equation (6.17), we can now explain strengthening mechanisms.

Solid solution hardening Solid solution hardening is strengthening by alloying with other elements that disperse atomically into the host lattice (Figure 6.20(a)). They are added when the alloy is molten and are then trapped in the solid lattice as it solidifies. Adding zinc to copper makes the alloy *brass* – copper dissolves up to 30% zinc. The zinc atoms replace copper atoms to form a random *substitutional solid solution*. The zinc atoms are bigger than those of copper, and, in squeezing into the copper lattice, they distort it. This 'roughens' the slip plane, so to speak, making it harder for dislocations to move – they are pinned by the solute atoms and bow out, thereby adding an additional resistance f_{ss} to dislocation motion. The figure illustrates that the concentration of solute, c, expressed as an atom fraction within a given plane, is on average:

$$c = \frac{b^2}{L^2}$$

where L is the spacing of obstacles in the slip plane, and b is the atom size. Thus,

$$L = \frac{b}{c^{1/2}}$$

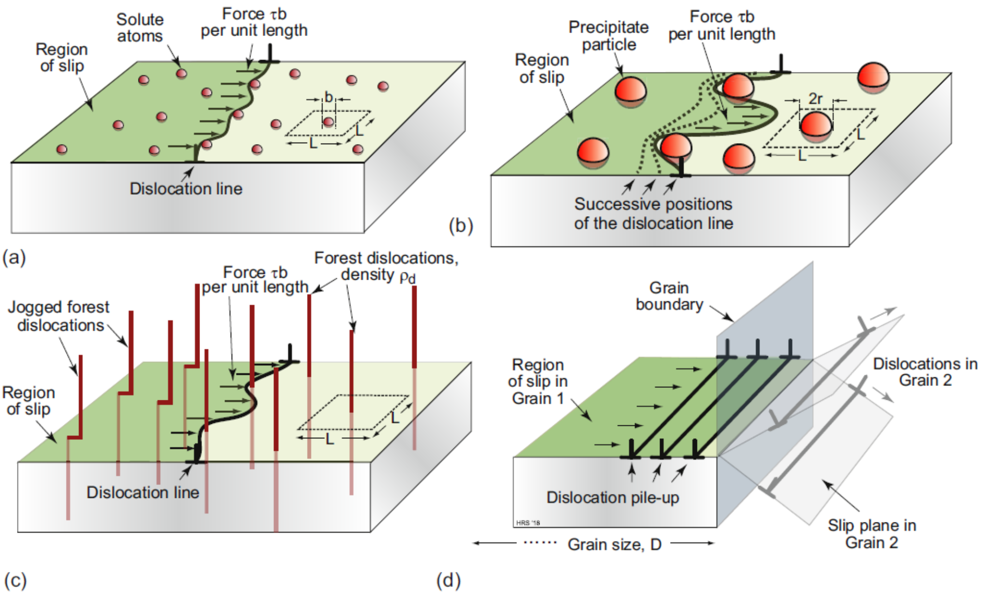


Figure 6.20 (a) Solid solution hardening. (b) Precipitation or dispersion hardening. (c) Forest hardening (work hardening). (d) Grain boundary hardening.

Plugging this into equation (6.17) relates the concentration of the solid solution to the additional shear stress τ_{ss} required to move the dislocation:

$$\tau_{ss} = \alpha G c^{1/2}. \tag{6.18}$$

So τ_{ss} increases as the square root of the solute concentration. Brass, bronze and stainless steels, and many other metallic alloys, derive their strength in this way. They differ only in the extent to which the solute distorts the crystal, described by the constant α. The bowing angle θ for solute is small – much less than 90° – so solute provides weak pinning.

Dispersion and precipitate strengthening A more effective way to impede dislocations is to disperse small, strong particles in their path. One way to make such a microstructure is to mix small solid particles of a high melting point compound into a liquid metal and to cast it to shape, trapping the particles in place – it is the way that metal–matrix composites such as Al–SiC are made. But in practice, these particles are rather far apart, so an alternative route is used to give the best outcomes – this is to form the particles *in situ* by a *precipitation* process, controlled by *heat treatment*. Solute atoms often form solid solutions more easily at high temperature than low. So if a solute is first dissolved in a metal at high temperature, and the alloy is cooled, the solute precipitates out as tiny crystals of compounds embedded in the host lattice – much as salt will crystallize from a saturated solution when it is cooled. An alloy of aluminium containing 4% copper, for instance, precipitates very small, closely spaced particles of the hard compound $CuAl_2$. Copper alloyed with a little beryllium, similarly treated, gives precipitates of the compound CuBe. And most steels are strengthened by carbide precipitates obtained in this way. In practical heat treatment, the particle spacing is kept as small as possible by first quenching to room temperature and then reheating to form the precipitates – this is discussed in more depth in *Guided Learning Unit 2*.

Figure 6.20(b) shows how particles obstruct dislocation motion. In this case, the particles are 'strong' obstacles, and the dislocation bows out to $\theta = 90°$, escaping as shown in Figure 6.19(c), leaving a loop of dislocation wrapped round the particle. In this semi-circular configuration, the dislocation of line tension $T = 1/2\ Gb^2$ is applying the biggest force it can on each obstacle, equal to $2T$, so this is the maximum pinning force p. From equations (6.15) and (6.16), the strength contribution from precipitates is therefore

$$\tau_{ppt} = \frac{2T}{bL} = \frac{Gb}{L} \qquad (6.19)$$

Comparison with equation (6.17) shows that the maximum value of the constant α is therefore 1. Precipitation hardening is the most effective way to increase strength: precipitation-hardened aluminium alloys can be 15 times stronger than pure aluminium.

Work hardening The rising part of the stress–strain curve of Figure 6.1 is known as *work hardening*: the plastic deformation causes dislocations to accumulate. The *dislocation density*, ρ_d, is defined as the length of dislocation line per unit volume (m/m^3). Even in an annealed soft metal, the dislocation density is around 10^{10} m/m^3, meaning that a 1-cm cube (the size of a cube of sugar) contains about 10 km of dislocation line. When metals are deformed, dislocations multiply, causing their density to grow to as much as 10^{17} m/m^3 or more – 100 million km per cubic centimetre! The reason that the yield stress rises is because a moving dislocation now finds that its slip plane is penetrated by a 'forest' of intersecting dislocations with a steadily decreasing average spacing $L = \rho_d^{-1/2}$ (since ρ_d is a number per unit area). Figure 6.20(c) shows the idea schematically. As a moving dislocation advances, it bows out between pinning points at dislocation intersections – a relatively weak obstacle ($\alpha < 1$). As the pinning point gives way, material above the slip plane slips relative to that below and creates a little step called a *jog* in each forest dislocation. These jogs have potential energy – they are tiny segments of dislocation of length b – and this is the source of the pinning force p, with the result that p again scales with $Gb^2/2$. Assembling these results into equation (6.17) gives the work-hardening contribution to the shear stress:

$$\tau_{wh} = \alpha \frac{Gb}{L} = \alpha Gb\sqrt{\rho_d} \tag{6.20}$$

The greater the density of dislocations, the smaller the spacing between them, and so the greater their contribution to τ_{wh}.

All metals work harden. It can be a nuisance: if you want to roll thin sheet, work hardening quickly raises the yield strength so much that you have to stop and *anneal* the metal (that is, heat it up to remove the accumulated dislocations) before you can go on – a trick known to blacksmiths for centuries. But it is also useful: it is a potent strengthening method, particularly for alloys that cannot be heat-treated to give precipitation hardening.

Grain boundary hardening Grain boundary hardening is a little different to the other mechanisms. Almost all metals are polycrystalline, made up of tiny, randomly oriented grains, meeting at grain boundaries like those of Figure 6.12(d). The grain size, D, is typically 10–100 μm. These boundaries obstruct dislocation motion, as shown in Figure 6.20(d). Dislocations in Grain 1 can't just slide into the next one (Grain 2) because the slip planes don't line up. Instead, new dislocations have to nucleate in Grain 2 with combined slip vectors that match the displacement caused by dislocations in Grain 1. The mechanism is as follows: dislocations 'pile up' at the boundary, creating the stress needed in the next grain to nucleate the matching dislocations. The number of dislocations in a pile-up scales with the size of the grain, and the bigger the pile-up, the lower the extra shear stress needed to enable slip to continue into the next grain. This gives another contribution to the shear stress, τ_{gb}, that is found to scale as $D^{-1/2}$, giving

$$\tau_{gb} = \frac{k_p}{\sqrt{D}} \tag{6.21}$$

where k_p is called the Hall-Petch constant after the scientists who first measured it. For normal grain sizes, τ_{gb} is small and not a significant source of strength, but for materials that are microcrystalline ($D < 1$ μm) or nanocrystalline (D approaching 1 nm), it becomes significant. Controlling the grain size is routine in conventional processing – in the initial casting of a liquid to make a solid shape, and then by deformation and heat treatments that lead to 'recrystallisation' in the solid – more on this in Chapter 19. More exotic processing is needed to produce really fine grain sizes – rapid cooling from the melt, electroplating, vapor deposition, laser surface treatment, or compaction of nanoscale particles. These are difficult to make and not easy to keep fine-grained – the energy per unit area associated with grain boundaries means that, given a little thermal energy, the grains grow.

Relationship between dislocation strength and yield strength To a first approximation the strengthening mechanisms add up, giving a *dislocation shear yield strength*, τ_y, of

$$\tau_y = \tau_i + \tau_{ss} + \tau_{ppt} + \tau_{wh} + \tau_{gb} \tag{6.22}$$

Strong materials either have a high intrinsic strength, τ_i (like diamond), or they rely on the superposition of solid solution strengthening τ_{ss}, precipitation τ_{ppt} and work hardening τ_{wh}

(like high-tensile steels). Nanocrystalline solids exploit, in addition, the contribution of τ_{gb}. But before we can use this information, one problem remains: we have calculated the shear yield strength of one crystal, loaded in shear *parallel to a slip plane*. What we want is the yield strength of a *polycrystalline material* in *tension*. To link them, there are two simple steps.

First, when a remote shear stress is applied to an aggregate of crystals, some crystals will have their slip planes oriented favourably with respect to the shear stress; others will not. This randomness of orientation means that the shear stress applied to a polycrystal that is needed to make *all* the crystals yield is higher than τ_y, by a factor of about 1.5 (called the Taylor factor, after the same G.I. Taylor who postulated the dislocation mechanism, with Orowan). This remote shear stress is referred to as the *yield strength in shear*, k – it is the shear stress that would cause a tube to yield in pure torsion, for instance.

Second, yield strength is measured in uniaxial tension, so how does this axial stress produce the shear stress k that leads to yield of the polycrystal? The answer is that a uniform tensile stress σ creates shear stress on all planes that lie at an angle to the tensile axis. Figure 6.21 shows a tensile force F acting on a rod of cross-section A. An inclined plane is shown with the normal to the plane at an angle θ to the loading direction. It carries components of force $F\sin\theta$ and $F\cos\theta$, parallel and perpendicular to the plane. The area of this plane is $A/\cos\theta$, so the shear stress parallel to the plane is

$$\tau = \frac{F\sin\theta}{A/\cos\theta} = \sigma\sin\theta\cos\theta$$

where $\sigma = F/A$ is the tensile stress. The variation of τ is plotted against θ in the figure – the maximum shear stress is found on planes at an angle of 45°, when $\tau = \sigma/2$. The yield strength in shear k is therefore equal to $\sigma_y/2$, where σ_y is the tensile yield stress.

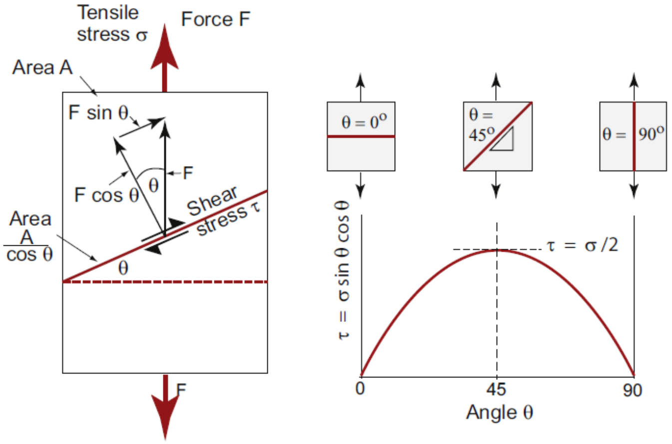


Figure 6.21 The variation of stress with angle of the reference axes. A tensile stress σ gives a maximum shear stress $\tau = \sigma/2$ on a plane at 45° to the tensile axis.

Combining these two factors, the tensile stress to cause yielding of a polycrystalline sample is approximately three times the shear strength of a single crystal:

$$\sigma_y = 2k \approx 3\tau_y$$

As a result, the superposition of strengthening mechanisms in equation (6.22) applies equally to the yield strength, σ_y.

Strength and ductility of alloys Of all the properties that materials scientists and engineers have sought to manipulate, the strength of metals and alloys is probably the most explored. It is easy to see why: Table 6.1 gives a small selection of the applications of metals and their alloys – their importance in engineering design is clearly enormous. The hardening mechanisms are often used in combination. This is illustrated graphically for copper alloys in Figure 6.22. Good things, however, have to be paid for. Here the payment for increased strength is, almost always, loss of ductility, so the elongation ε_f is reduced. The material is stronger, but it cannot be deformed as much without fracture. This general trend is evident in Figure 6.23, which shows the nominal stress–strain curves for a selection of engineering alloys. Chapter 19 and *Guided Learning Unit 2* will explore the practicalities of manipulating the strength and ductility of alloys in the context of processing.

Table 6.1 Metal alloys with typical applications, indicating the strengthening mechanisms used

Alloy	Typical uses	Solution hardening	Precipitation hardening	Work hardening
Pure Al	Kitchen foil			✓✓✓
Pure Cu	Wire			✓✓✓
Cast Al, Mg	Automotive parts	✓✓✓	✓	
Bronze (Cu–Sn), Brass (Cu–Zn)	Marine components	✓✓✓	✓	✓✓
Non-heat-treatable wrought Al	Ships, cans, structures	✓✓✓	✓✓✓	
Heat-treatable wrought Al	Aircraft, structures	✓	✓✓✓	✓
Low-carbon steels	Car bodies, structures, ships, cans	✓✓✓	✓✓✓	
Low-alloy steels	Automotive parts, tools	✓	✓✓✓	✓
Stainless steels	Pressure vessels	✓✓✓	✓	✓✓✓
Cast Ni alloys	Jet engine turbines	✓✓✓	✓✓✓	

Symbols: ✓✓✓, Routinely used; ✓, Sometimes used.

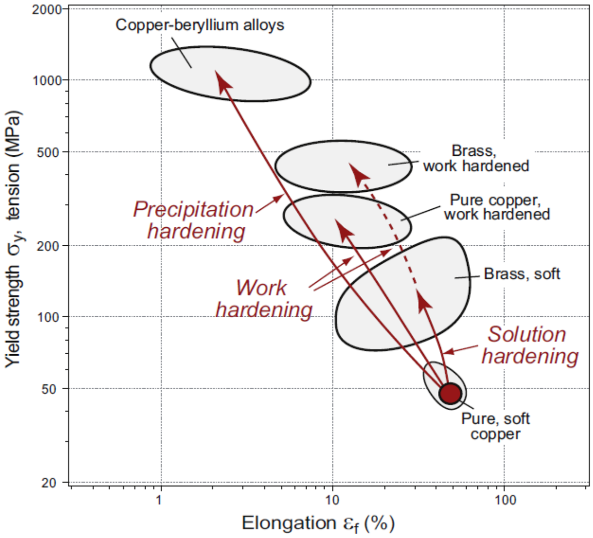


Figure 6.22 Strengthening and ductility (elongation to fracture) for copper alloys. The hardening mechanisms are frequently combined, but in general, the greater the strength, the lower the ductility.

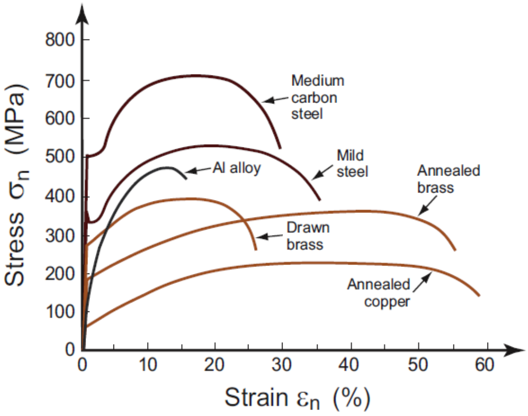


Figure 6.23 Nominal stress–strain curves for a selection of engineering alloys. Stronger alloys tend to have lower ductility.

Strengthening polymers In non-crystalline solids, the dislocation is not a helpful concept. We think instead of some unit step of the flow process: the relative slippage of two segments of a polymer chain, or the shear of a small molecular cluster in a glass network. Their strength has the same origin as that underlying the lattice resistance: if the unit step involves breaking strong bonds (as in an inorganic glass), the materials will be strong and brittle, as ceramics are. If it only involves the rupture of weak bonds (the van der Waals bonds in polymers, for example), it will have low strength. Polymers, too, must therefore be strengthened by impeding the slippage of segments of their molecular chains. This is achieved by *blending*, by *drawing*, by *cross-linking* and by *reinforcement* with particles, fibres or fabrics – again, we manipulate the microstructure-sensitive property of strength by a combination of composition and processing (see Chapter 19 for more details).

A blend is a mixture of two polymers, stirred together in a sort of industrial food mixer. The strength and modulus of a blend are just the average of those of the components, weighted by volume fraction (a rule of mixtures again). If one of these is a low molecular weight hydrocarbon, it acts as a plasticiser, reducing the modulus and giving the blend a leather-like flexibility.

Drawing is the deliberate use of the molecule-aligning effect of stretching, like that sketched in Figure 6.11(a), to greatly increase stiffness and strength in the direction of stretch. Fishing line is drawn nylon, mylar film is a polyester with molecules aligned in the plane of the film, and geotextiles, used to restrain earth banks, are made from drawn polypropylene.

Cross-linking, sketched in Figures 4.36 and 4.37, creates strong bonds between molecules that were previously linked by weak van der Waals forces. Vulcanized rubber is rubber that has been cross-linked, and the superior strength of epoxies derives from cross-linking.

Reinforcement is possible with particles of cheap fillers – sand, talc or wood dust. Far more effective is reinforcement with fibres – usually glass or carbon – either continuous or chopped, as explained in Chapter 4.

6.6 Summary and conclusions

Load-bearing structures require materials with reliable, reproducible strength. There is more than one measure of strength. *Elastic design* requires that no part of the structure suffers plastic deformation, and this means that the stresses in it must nowhere exceed the *yield strength*, σ_y, of ductile materials or the *elastic limit* of those that are not ductile. *Plastic design* and *deformation processing*, by contrast, require some or all parts of the structure to deform plastically, either to absorb energy or to enable the shape to be changed. Then two further properties become relevant: the *ductility*, ε_f, and the tensile strength, σ_{ts}, which are the maximum strain and the maximum stress the material can tolerate before fracture. The tensile strength is generally larger than the yield strength because of *work hardening*.

Charts plotting strength, like those plotting modulus, show that material families occupy different areas of material property space, depending on the strengthening mechanisms on which they rely. Strength for metals is a particularly microstructure-sensitive property. Crystal defects – particularly *dislocations* – are central to the understanding of alloy hardening. It is the motion of dislocations that gives plastic flow in crystalline solids, giving them unexpectedly low strengths. When strength is needed, it has to be provided by the strengthening mechanisms that impede dislocation motion.

First among these is the *lattice resistance* – the intrinsic resistance of the crystal to dislocation motion. Others can be deliberately introduced by alloying and heat treatment. *Solid solution hardening*, *dispersion* and *precipitation hardening*, *work hardening* and *grain boundary hardening* add to the lattice resistance. The strongest materials utilise more than one mechanism.

Non-crystalline solids – particularly polymers – deform in a less organised way by the tangled polymer chains sliding and sometimes aligning with the direction of deformation. This leads to *cold drawing* with substantial plastic strain and, at lower temperatures, to *crazing* or *shear banding*. The stress required to do this is significant, giving polymers useful intrinsic strength. This can be enhanced by blending, cross-linking and reinforcement with particles or fibres to give the engineering polymers we use today.